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Some Symmetries in Theories with Higher Derivatives

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Symmetries are examined in theories with higher derivatives and both ordinary and Grassmann independent variables. The field equations are established. The general variation of the action is performed. Spatiotemporal invariance is studied. Internal symmetries and conservation laws in classical and quantum fields are discussed.

1. INTRODUCTION

One of the lines of interest concerning the enlargement of field theories was the introduction of higher derivatives. Since the work of Bopp(1) and Podolsky(2) generalizing electrodynamics, theories with higher derivatives have been used in different domains. Among these the introduction of higher derivatives in SUSY theories have been proposed (e.g., refs. 3–5). On the other hand, in SUSY theories auxiliary independent variables are introduced. In the present paper we view both aspects and examine symmetries in theories in which the Lagrangian contains two classes of independent variables and also second-order derivatives. With this Lagrangian we define the action by

$$A = \int d\theta \cdot dx\, L(x_\lambda, \theta_\alpha, \varphi_k, \partial_\lambda \varphi_k, \partial_\alpha \varphi_k, \partial^2_{\lambda\mu} \varphi_k, \partial^2_{\alpha\beta} \varphi_k) \qquad (1.1)$$

L is the Lagrangian "density," x_λ are the spacetime variables, θ_α are the supplementary independent variables, and φ_k are the generalized coordinates (fields).

The following notations have been used:

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$$dx = \prod_{\lambda} dx_\lambda, \qquad d\theta = \prod_{\alpha} d\theta_\alpha$$
$$\partial_\lambda = \partial/\partial x_\lambda, \qquad \partial_\alpha = \partial/\partial\theta_\alpha \tag{1.2}$$
$$\partial^2_{\lambda\mu} = \partial^2/\partial x_\lambda\, \partial x_\mu, \qquad \partial^2_{\alpha\beta} = \partial^2/\partial\theta_\alpha\, \partial\theta_\beta$$

2. FIELD EQUATIONS

As first step we assume the action principle of the form

$$\delta_0 A = \delta_0 \int d\theta\, dx\, L = \int d\theta \int dx\, \delta_0 L = 0 \tag{2.1}$$

with fixed boundary for all integrals.

We define the variation

$$\delta_0 L = L(\varphi_k + \delta_0\varphi_k, \partial_\lambda\varphi_k + \delta_0\partial_\lambda\varphi_k, \partial_\alpha\varphi_k + \delta_0\partial_\alpha\varphi_k, \partial^2_{\lambda\mu}\varphi_k + \delta_0\partial^2_{\lambda\mu}\varphi_k,$$
$$\partial^2_{\alpha\beta}\varphi_k + \delta_0\partial^2_{\alpha\beta}\varphi_k) - L(\varphi_k, \partial_\lambda\varphi_k, \partial_\alpha\varphi_k, \partial^2_{\lambda\mu}\varphi_k, \partial^2_{\alpha\beta}\varphi_k) \tag{2.2}$$

Applying Taylor's expansion to the first term on the right-hand side in (2.2) and keeping only first-order terms, we obtain

$$\delta_0 L = \frac{\partial L}{\partial\varphi_k}\delta_0\varphi_k + \frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\delta_0\partial_\lambda\varphi_k + \frac{\partial L}{\partial(\partial_\alpha\varphi_k)}\delta_0\partial_\alpha\varphi_k$$
$$+ \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\delta_0\partial^2_{\lambda\mu}\varphi_k + \frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)}\delta_0\partial^2_{\alpha\beta}\varphi_k = 0 \tag{2.3}$$

With some lengthy calculations Eq. (2.3) can be put into the form

$$\delta_0 A = \int d\theta \int dx \Bigg\{ \frac{\partial L}{\partial\varphi_k}\delta_0\varphi_k - \left(\partial_\lambda \frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\right)\delta_0\varphi_k + \left(\partial^2_{\lambda\mu}\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right)\delta_0\varphi_k$$
$$+ \partial_\lambda\left(\frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\delta_0\varphi_k\right) + \partial_\lambda\left(\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\partial_\mu\delta_0\varphi_k\right)$$
$$- \partial_\lambda\left[\left(\partial_\mu\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right)\delta_0\varphi_k\right] - \left(\partial_\alpha\frac{\partial L}{\partial(\partial_\alpha\varphi_k)}\right)\delta_0\varphi_k$$
$$+ \left(\partial^2_{\alpha\beta}\frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)}\right)\delta_0\varphi_k + \partial_\alpha\left(\frac{\partial L}{\partial(\partial_\alpha\varphi_k)}\delta_0\varphi_k\right)$$
$$+ \partial_\alpha\left(\frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)}\partial_\beta\delta_0\varphi_k\right) - \partial_\alpha\left[\left(\partial_\beta\frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)}\right)\delta_0\varphi_k\right]\Bigg\} = 0 \tag{2.4}$$

Since the boundary is fixed, one can see that the terms 4–6 and 9–11 vanish by integration, and since the variations $\delta_0\varphi_k$ are arbitrary, the remaining terms yield the equations

$$\frac{\partial L}{\partial \varphi_k} - \partial_\lambda \frac{\partial L}{\partial(\partial_\lambda \varphi_k)} + \partial^2_{\lambda\mu} \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} - \partial_\alpha \frac{\partial L}{\partial(\partial_\alpha \varphi_k)} + \partial^2_{\alpha\beta} \frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)} = 0 \tag{2.5}$$

Considering θ_α as Grassmann variables, Eq. (2.5) yields the field equations in superspace with higher derivatives.

Dropping the second-order derivatives, one obtains the usual field equations in superspace, for instance,[6]

$$\frac{\partial L}{\partial \varphi_k} - \frac{\partial}{\partial \theta} \frac{\partial L}{\partial(\partial \varphi_k / \partial \theta)} - \frac{\partial}{\partial \bar{\theta}} \frac{\partial L}{\partial(\partial \varphi_k / \partial \bar{\theta})} - \frac{\partial}{\partial t} \frac{\partial L}{\partial(\partial \varphi_k / \partial t)} = 0 \tag{2.6}$$

3. GENERAL VARIATION OF THE ACTION

We compute a general variation of the action, including variations of the boundary of the integrals over x_λ variables:

$$\delta A = \delta \int d\theta \int dx\ L = \int d\theta \cdot \delta \int dx\ L \tag{3.1}$$

where

$$\delta \int dx\ L = \int (\delta\ dx)\ L + \int dx\ \delta L \tag{3.2}$$

and

$$\int (\delta\ dx) L = \int dx\ (\partial_\lambda \delta x_\lambda)\ L = \int dx\ [\partial_\lambda (L \delta x_\lambda) - (\partial_\lambda L)\ \delta x_\lambda)] \tag{3.3}$$

The variation of L is given by

$$\delta L = \frac{\partial L}{\partial \varphi_k} \delta\varphi_k + \frac{\partial L}{\partial(\partial_\lambda \varphi_k)} \delta\partial_\lambda\varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \delta\partial^2_{\lambda\mu}\varphi_k + \frac{\partial L}{\partial(\partial_\alpha \varphi_k)} \delta\partial_\alpha\varphi_k + \frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)} \delta\partial^2_{\alpha\beta}\varphi_k \tag{3.4}$$

The increments $\delta\varphi_k$ are defined as follows. Denote

$$\varphi'_k(x_\lambda, \theta_\alpha) = \varphi_k(x_\lambda, \theta_\alpha) + \delta_0\varphi_k \tag{3.5}$$

The increment $\delta\varphi_k$ is the difference

$$\delta\varphi_k = \varphi'_k(x_\lambda + \delta x_\lambda, \theta_\alpha + \delta\theta_\alpha) - \varphi_k(x_\lambda, \theta_\alpha) \tag{3.6}$$

Applying the Taylor expansion to φ'_k and dropping the higher terms, one obtains

$$\varphi'_k(x_\lambda + \delta x_\lambda, \theta_\alpha + \delta\theta_\alpha) = \varphi_k(x_\lambda, \theta_\alpha) + \delta_0\varphi_k + [\partial_\lambda\varphi'_k(x_\lambda, \theta_\alpha)]\, \delta x_\lambda + [\partial_\alpha\varphi'_k(x_\lambda, \theta_\alpha)]\, \delta\theta_\alpha \tag{3.7}$$

In view of (3.6) we have

$$\delta\varphi_k = \delta_0\varphi_k + (\partial_\lambda\varphi_k)\, \delta x_\lambda + (\partial_\alpha\varphi_k)\, \delta\theta_\alpha \tag{3.8}$$

Similarly we obtain

$$\delta\partial_\lambda\varphi_k = \delta_0\partial_\lambda\varphi_k + (\partial^2_{\xi x}\varphi_k)\, \delta x_\xi + (\partial^2_{\varepsilon\alpha}\varphi_k)\, \delta\theta_\varepsilon \tag{3.9}$$

$$\delta\partial_\alpha\varphi_k = \delta_0\partial_\alpha\varphi_k + (\partial^2_{\xi\alpha}\varphi_k)\, \delta x_\xi + (\partial^2_{\varepsilon\alpha}\varphi_k)\, \delta\theta_\varepsilon \tag{3.10}$$

$$\delta\partial^2_{\lambda\mu}\varphi_k = \delta_0\partial^2_{\lambda\mu}\varphi_k + (\partial^3_{\xi\lambda\mu}\varphi_k)\delta x_\xi + (\partial^3_{\varepsilon\lambda\mu}\varphi_k)\, \delta\theta_\varepsilon \tag{3.11}$$

$$\delta\partial^2_{\alpha\beta}\varphi_k = \delta_0\partial^2_{\alpha\beta}\varphi_k + (\partial^3_{\xi\alpha\beta}\varphi_k)\delta x_\xi + (\partial^3_{\varepsilon\alpha\beta}\varphi_k)\, \delta\theta_\varepsilon \tag{3.12}$$

Now, bearing in mind (3.2)–(3.4) and (3.8)–(3.12), the variation (3.1) reads

$$\begin{aligned}
\delta A = \int d\theta \int dx \Bigg[& \partial_\lambda(L\delta x_\lambda) - (\partial_\lambda L)\, \delta x_\lambda + \frac{\partial L}{\partial\varphi_k}\, \delta_0\varphi_k \\
& + \frac{\partial L}{\partial(\partial_\alpha\varphi_k)}\, \delta_0\partial_\alpha\varphi_k + \frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)}\, \delta_0\partial^2_{\alpha\beta}\varphi_k + \frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\, \delta_0\partial_\lambda\varphi_k \\
& + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\, \delta_0\partial^2_{\lambda\mu}\varphi_k + \frac{\partial L}{\partial\varphi_k}\, (\partial_\alpha\varphi_k)\, \delta\theta_\alpha + \frac{\partial L}{\partial(\partial_\alpha\varphi_k)}\, (\partial^2_{\varepsilon\alpha}\varphi_k)\, \delta\theta_\varepsilon \\
& + \frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)}\, (\partial^3_{\varepsilon\alpha\beta}\varphi_k)\, \delta\theta_\varepsilon + \frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\, (\partial^2_{\varepsilon\lambda}\varphi_k)\, \delta\theta_\varepsilon \\
& + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\, (\partial^3_{\varepsilon\lambda\mu}\varphi_k)\, \delta\theta_\varepsilon + \frac{\partial L}{\partial\varphi_k}\, (\partial_\lambda\varphi_k)\, \delta x_\lambda \\
& + \frac{\partial L}{\partial(\partial_\alpha\varphi_k)}\, (\partial^2_{\xi\alpha}\varphi_k)\, \delta x_\xi + \frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)}\, (\partial^3_{\xi\alpha\beta}\varphi_k)\, \delta x_\xi
\end{aligned}$$

$$+ \frac{\partial L}{\partial(\partial_\lambda \varphi_k)} (\partial^2_{\xi\lambda}\varphi_k)\, \delta x_\xi + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} (\partial^3_{\xi\lambda\mu}\varphi_k)\, \delta x_\xi \Bigg] \tag{3.13}$$

In Eq. (3.13) terms 3–7 in the brackets yield the expression (2.4). Thus since the boundary of the integrals over θ_α is fixed, the corresponding terms vanish by integration, and bearing in mind Eq. (2.5), the only remaining terms are

$$\partial_\lambda \left[\frac{\partial L}{\partial(\partial_\mu \varphi_k)} \delta_0\varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \partial_\mu \delta_0 \varphi_k - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \right) \delta_0 \varphi_k \right] \tag{3.14}$$

Also the terms 8–12 in the brackets in (3.13) vanish by integration. The terms 13–17 are canceled by the expression

$$\begin{aligned}(\partial_\xi L)\delta x_\xi = {} & L_{|\xi}\, \delta x_\xi + \frac{\partial L}{\partial \varphi_k} (\partial_\xi \varphi_k)\, \delta x_\xi + \frac{\partial L}{\partial(\partial_\alpha \varphi_k)} (\partial^2_{\xi\alpha}\varphi_k)\, \delta x_\xi \\ & + \frac{\partial L}{\partial(\partial^2_{\alpha\beta}\varphi_k)} (\partial^3_{\xi\alpha\beta}\varphi_k)\, \delta x_\xi + \frac{\partial L}{\partial(\partial_\lambda \varphi_k)} (\partial^2_{\xi\lambda}\varphi_k)\, \delta x_\xi \\ & + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} (\partial^3_{\xi\lambda\mu}\varphi_k)\, \delta x_\xi \end{aligned} \tag{3.15}$$

with only $-L_{|\xi}\, \delta x_\xi$ remaining, where $L_{|\xi}$ is the explicit partial derivative. Thus (3.13) reduces to

$$\begin{aligned}\delta A = \int d\theta \int dx \Bigg\{ \partial_\lambda \Bigg[& L\, \delta x_\lambda + \frac{\partial L}{\partial(\partial_\lambda \varphi_k)} \delta_0 \varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \partial_\mu \delta_0 \varphi_k \\ & - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \right) \delta_0 \varphi_k \Bigg] - L_{|\xi}\, \delta x_\xi \Bigg\} \end{aligned} \tag{3.16}$$

Using the expressions (3.8)–(3.10), Eq. (3.16) becomes

$$\begin{aligned}\delta A = \int d\theta \int dx\, \partial_\lambda \Bigg\{ & \frac{\partial L}{\partial(\partial_\lambda \varphi_k)} \delta\varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \delta\partial_\mu \varphi_k - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \right) \delta\varphi_k \\ & - \left[\frac{\partial L}{\partial(\partial_\lambda \varphi_k)} \partial_\nu \varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \partial^2_{\mu\nu}\varphi_k - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \right) \partial_\nu \varphi_k \right. \\ & \left. - L\, \delta^{\lambda\nu} \right] \delta x_\nu - \left[\frac{\partial L}{\partial(\partial_\lambda \varphi_k)} \partial_\alpha \varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \partial^2_{\mu\alpha}\varphi_k \right. \end{aligned}$$

$$- \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \right) \partial_\alpha \varphi_k \Bigg] \delta\theta_\alpha \Bigg\} \tag{3.17}$$

(L is explicitly independent of x_λ).

4. SPACE-TIME INVARIANCE

In order to examine symmetry transformations leaving unchanged the physical system, we seek the invariance of the action under different transformations. First consider space-time infinitesimal transformations. In that case only transformations δx_λ occur in (3.17). One can then write

$$\delta A = \int d\theta \int dx \Bigg(- \partial_\lambda \Bigg\{ \Bigg[\frac{\partial L}{\partial(\partial_\lambda \varphi_k)} \partial_\nu \varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \partial^2_{\mu\nu}\varphi_k - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \right) \partial_\nu \varphi_k - L\, \delta^{\lambda\nu} \Bigg] \delta x_\nu \Bigg\} \Bigg) = 0 \tag{4.1}$$

which leads to

$$\partial_\lambda \mathcal{T}^{\lambda\nu} \tag{4.2}$$

namely the conservation of the quantity

$$\begin{aligned} \mathcal{T}^{\lambda\nu} &= \frac{\partial L}{\partial(\partial_\lambda \varphi_k)} \partial_\nu \varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \partial^2_{\mu\nu}\varphi_k - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \right) \partial_\nu \varphi_k - L\, \delta^{\lambda\nu} \\ &= T^{\lambda\nu} + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \partial^2_{\mu\nu}\varphi_k - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)} \right) \partial_\nu \varphi_k \end{aligned} \tag{4.3}$$

$T^{\lambda\nu}$ is the usual energy-momentum tensor; therefore one can call $\mathcal{T}^{\lambda\nu}$ the generalized energy-momentum tensor of the present theory.

Let us now write the elementary variations of the independent variables by the linear relations

$$\delta x_\lambda = e_{\lambda\mu} x_\mu + e_\lambda \tag{4.4}$$

with

$$e_{\lambda\mu} + e_{\mu\lambda} = 0 \tag{4.5}$$

where e are first-order infinitesimals. Inserting (4.4) in (4.2) given by (4.1), one obtains

$$\partial_\lambda \left[-\mathcal{T}^{\lambda\nu}(e_{\nu\xi}x_\xi + e_\nu)\right] = \partial_\lambda\left[-\frac{1}{2}(\mathcal{T}^{\lambda\nu}x_\xi - \mathcal{T}^{\xi\lambda}x_\nu)e_{\nu\xi} - \mathcal{T}^{\lambda\nu}e_\nu\right]$$
$$= -\frac{1}{2}[\partial_\lambda(\mathcal{T}^{\lambda\nu}x_\xi - \mathcal{T}^{\xi\lambda}x_\nu)]e_{\nu\xi} - (\partial_\lambda\mathcal{T}^{\lambda\nu})e_\nu = 0 \quad (4.6)$$

If only transformations e_ν occur, this leads to the conservation law (4.2). If separate transformations $e_{\nu\xi}$ occur, then one obtains the conservation of the quantity

$$\mathcal{R}^{\lambda\nu\xi} = \mathcal{T}^{\lambda\nu}x_\xi - \mathcal{T}^{\xi\lambda}x_\nu \quad (4.7)$$

Observing that in the absence of higher derivatives $\mathcal{R}^{\lambda\nu\xi}$ reduces to the "orbital" angular momentum tensor of the usual field theory, one is justified in interpreting (4.7) as the generalized "orbital" angular momentum tensor of the present theory.

If only $\delta\theta_\alpha$ transformations occur, the invariance of the action leads to

$$\delta A = \int d\theta \int dx\left\{-\partial_\lambda\left[\frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\,\partial_\alpha\varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\,\partial^2_{\mu\alpha}\varphi_k - \left(\partial_\mu\,\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right)\partial_\alpha\varphi_k\right]\right\} = 0 \quad (4.8)$$

The interpretation of this law needs a special study, perhaps within the SUSY theory, but this is beyond the scope of this paper.

5. INTERNAL SYMMETRY IN CLASSICAL FIELDS

Consider a classical complex field φ_k and phase transformations of the form

$$\begin{aligned}\varphi_k &\to \varphi_k' = \exp(i\varepsilon)\,\varphi_k\\ \varphi_k^* &\to \varphi_k^{*\prime} = \exp(-i\varepsilon)\,\varphi_k^*\end{aligned} \quad (5.1)$$

where ε is a real arbitrary constant. We have

$$\begin{aligned}\exp(i\varepsilon) &= 1 + i\varepsilon + (i^2/2!)\,\varepsilon^2 + \dots\\ \exp(-i\varepsilon) &= 1 - i\varepsilon + (i^2/2!)\,\varepsilon^2 - \dots\end{aligned} \quad (5.2)$$

and for infinitesimal transformations

$$\begin{aligned}\exp(i\varepsilon) &= 1 + i\varepsilon\\ \exp(-i\varepsilon) &= 1 - i\varepsilon\end{aligned} \quad (5.3)$$

We then can write

$$\begin{aligned} \varphi'_k &= \varphi_k + i\varepsilon\varphi_k \\ \varphi_k^{*'} &= \varphi_k^* - i\varepsilon\varphi_k^* \end{aligned} \tag{5.4}$$

whence

$$\begin{aligned} \delta\varphi_k &= i\varepsilon\varphi_k \\ \delta\varphi_k^* &= -i\varepsilon\varphi_k^* \end{aligned} \tag{5.5}$$

and

$$\begin{aligned} \partial_\lambda\delta\varphi_k &= i\varepsilon\partial_\lambda\varphi_k \\ \partial_\lambda\delta\varphi_k^* &= -i\varepsilon\partial_\lambda\varphi_k^* \end{aligned} \tag{5.6}$$

We require the action to be invariant under the above transformations. From (3.17), with $\delta x_\lambda = \delta\theta_\alpha = 0$, we have in this case

$$\begin{aligned} \partial_\lambda\Bigg[&\frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\,\delta\varphi_k + \frac{\partial L}{\partial(\partial_\lambda\varphi_k^*)}\,\delta\varphi_k^* + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\,\partial_\mu\delta\varphi_k \\ &+ \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k^*)}\,\partial_\mu\delta\varphi_k^* - \left(\partial_\mu\,\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right)\delta\varphi_k \\ &- \left(\partial_\mu\,\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k^*)}\right)\delta\varphi_k^*\Bigg] = 0 \end{aligned} \tag{5.7}$$

Introducing (5.5) and (5.6) in (5.7), we obtain

$$\begin{aligned} \partial_\lambda\Bigg[&\frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\,i\varepsilon\varphi_k + \frac{\partial L}{\partial(\partial_\lambda\varphi_k^*)}\,(-i\varepsilon)\varphi_k^* \\ &+ \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\,i\varepsilon\partial_\mu\varphi_k + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k^*)}\,(-i\varepsilon)\partial_\mu\varphi_k^* \\ &-\left(\partial_\mu\,\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right)i\varepsilon\varphi_k - \left(\partial_\mu\,\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k^*)}\right)(-i\varepsilon)\varphi_k^*\Bigg] = 0 \end{aligned} \tag{5.8}$$

One can write

$$\partial_\lambda\,\mathcal{J}_\lambda = 0 \tag{5.9}$$

indicating the conservation of the magnitude

$$\mathcal{J}_\lambda = i\varepsilon\left[\frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\varphi_k - \frac{\partial L}{\partial(\partial_\lambda\varphi_k^*)}\varphi_k^* + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\partial_\mu\varphi_k - \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k^*)}\partial_\mu\varphi_k^* - \left(\partial_\mu\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right)\varphi_k + \left(\partial_\mu\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k^*)}\right)\varphi_k^*\right] \tag{5.10}$$

which we interpret as the four-vector current in the present theory. The expression

$$Q = -i\int dV\,\mathcal{J}_4 \tag{5.11}$$

is the total charge. For real fields $\varphi_k = \varphi_k^*$, and thus real fields are neutral.

In the absence of the second-order derivatives $\mathcal{J}_\lambda$ reduces to the usual four-vector current. Thus one can write

$$\mathcal{J}_\lambda = j_\lambda + j_\lambda^S \tag{5.12}$$

where

$$j_\lambda = i\varepsilon\left[\frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\varphi_k - \frac{\partial L}{\partial(\partial_\lambda\varphi_k^*)}\varphi_k^*\right] \tag{5.13}$$

$$j_\lambda^S = i\varepsilon\left[\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\partial_\mu\varphi_k - \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k^*)}\partial_\mu\varphi_k^* - \left(\partial_\mu\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right)\varphi_k + \left(\partial_\mu\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k^*)}\right)\varphi_k^*\right] \tag{5.14}$$

6. INTERNAL SYMMETRY IN QUANTUM FIELDS

In quantum theories one has to examine invariance under the unitary transformation

$$UU^+ = U^+U = 1, \qquad U^+ = U^{-1} \tag{6.1}$$

a representation of which is given by

$$U = \exp i\varepsilon\Lambda_r G_r \tag{6.2}$$

where G_r are generators of a Lie group.

For infinitesimal transformations

$$U = 1 + i\varepsilon\Lambda_r G_r \tag{6.3}$$

The transformation of a quantum field φ_k is given by

$$\begin{aligned}\varphi_k \to \varphi_k' &= (1 + i\varepsilon\Lambda_r G_r)\varphi_k(1 - i\varepsilon\Lambda_r G_r)\\ &= \varphi_k + i\varepsilon\Lambda_r[G_r, \varphi_k]\end{aligned} \tag{6.4}$$

Denoting

$$[G_r, \varphi_k] = (M_r)_{kl}\varphi_l \tag{6.5}$$

one can write

$$\delta\varphi_k = i\varepsilon\Lambda_r(M_r)_{kl}\varphi_l \tag{6.6}$$

The Lagrangean must be invariant under transformations (6.3). Therefore (following ref. 7) we write

$$\begin{aligned}\delta L = \frac{\partial L}{\partial\Lambda_r}\,\delta\Lambda_r &= \frac{\partial L}{\partial\varphi_k}\frac{\delta\varphi_k}{\delta\Lambda_r}\,\delta\Lambda_r + \frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\,\frac{\delta(\partial_\lambda\varphi_k)}{\delta\Lambda_r}\,\delta\Lambda_r\\ &\quad + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\,\frac{\delta(\partial^2_{\lambda\mu}\varphi_k)}{\delta\Lambda_r}\,\delta\Lambda_r = 0\end{aligned} \tag{6.7}$$

But we have

$$\frac{\delta\varphi_k}{\delta\Lambda_r} = i\varepsilon(M_r)_{kl}\varphi_l \tag{6.8}$$

$$\frac{\delta(\partial_\lambda\varphi_k)}{\delta\Lambda_r} = \partial_\lambda\,\frac{\delta\varphi_k}{\delta\Lambda_r} = \partial_\lambda i\varepsilon(M_r)_{kl}\varphi_k \tag{6.9}$$

$$\frac{\delta(\partial^2_{\lambda\mu}\varphi_k)}{\delta\Lambda_r} = \partial^2_{\lambda\mu}\,\frac{\delta\varphi_k}{\delta\Lambda_r} = \partial^2_{\lambda\mu}\; i\varepsilon(M_r)_{kl}\varphi_l \tag{6.10}$$

Introducing (6.8)–(6.10) into (6.7), one obtains

$$\begin{aligned}&\frac{\partial L}{\partial_\lambda\varphi_k}\, i\varepsilon(M_r)_{kl}\varphi_l + \frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\,\partial_\lambda i\varepsilon(M_r)_{kl}\varphi_l\\ &\qquad + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\,\partial^2_{\lambda\mu}\; i\varepsilon(M_r)_{kl}\varphi_l\\ &= \frac{\partial L}{\partial_\lambda\varphi_k}\, i\varepsilon(M_r)_{kl}\varphi_l + \partial_\lambda\left(\frac{\partial L}{\partial(\partial_\lambda\varphi_k)}\; i\varepsilon(M_r)_{kl}\varphi_l\right)\end{aligned}$$

$$
\begin{aligned}
&-\left(\partial_\lambda \frac{\partial L}{\partial(\partial_\lambda \varphi_k)}\right) i\varepsilon (M_r)_{kl}\varphi_l - \partial_\lambda \left(\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\, \partial_\mu i\varepsilon (M_r)_{kl}\varphi_l\right) \\
&- \partial_\mu \left[\left(\partial_\lambda \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right) i\varepsilon (M_r)_{kl}\varphi_l\right] \\
&+ \left(\partial^2_{\lambda\mu} \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right) i\varepsilon (M_r)_{kl}\varphi_l = 0
\end{aligned}
\tag{6.11}
$$

In view of Eq. (1.5), Eq. (6.11) reads

$$
\begin{aligned}
\partial_\lambda \Bigg[& \frac{\partial L}{\partial(\partial_\lambda \varphi_k)}\, i\varepsilon (M_r)_{kl}\varphi_l + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\, \partial_\mu i\varepsilon (M_r)_{kl}\varphi_l \\
& - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right) i\varepsilon (M_r)_{kl}\varphi_l \Bigg] = 0
\end{aligned}
\tag{6.12}
$$

We interpret the quantity

$$
\begin{aligned}
\mathcal{J}^r_\lambda = i\varepsilon \Bigg[& \frac{\partial L}{\partial(\partial_\lambda \varphi_k)}\, (M_r)_{kl}\varphi_l + \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\, \partial_\mu (M_r)_{kl}\varphi_l \\
& - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right) (M_r)_{kl}\varphi_l \Bigg]
\end{aligned}
\tag{6.13}
$$

as the four-vector current of the present theory, and (6.12) represent its conservation. $\mathcal{J}^r_\lambda$ is made up of two parts: the usual four-vector current

$$
j^r_\lambda = i\varepsilon \frac{\partial L}{\partial(\partial_\lambda \varphi_k)}\, (M_r)_{kl}\varphi_l \tag{6.14}
$$

and the supplementary four-vector current

$$
j^{r(S)}_\lambda = i\varepsilon \left[\frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\, \partial_\mu (M_r)_{kl}\varphi_l - \left(\partial_\mu \frac{\partial L}{\partial(\partial^2_{\lambda\mu}\varphi_k)}\right) (M_r)_{kl}\varphi_l \right] \tag{6.15}
$$

Of course, as in the classical case, there is a total charge and a supplementary charge added to the usual one.

7. CONCLUSIONS

The goal of this paper was to examine some symmetries in theories based upon an action containing higher derivatives and two kind of independent variables, such as SUSY theories. To our knowledge, this study is new.

Some general conservation laws were inferred. Such general results could give indications about possible new specific conservation laws.

The advent of unified gauge theories had led physicists in recent years to question the absoluteness of many known conservation laws. For instance, the possibility that conservation of electric charge may break down has been discussed by some authors, (e.g., refs. 8 and 9). In this paper the four-vector current gets a supplementary term, beyond the usual one. This result can be interpreted as indicating that in general the total charge is conserved, but the usual charge only in some cases, when the supplementary term vanishes or is negligible.

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